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Do Dice Remember?

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We discuss the question of the existence of hidden variables within the formalism of Accardi and Fedullo (1982). In particular, we introduce a new condition at the level of hidden variable theories that we show to be sufficient in order to obtain non-Kolmogorovian probabilities. We compare this condition with Aerts' (1986) and Czachor's (1992) conditions, and propose a new kind of experimental test aimed at revealing the existence of hidden variables.

INTRODUCTION

The existence of a Kolmogorovian model for probabilities can be expressed by inequalities which constrict the space of Kolmogorovian, classical, probabilities.[2] It can be shown that the quantum probabilities violate these inequalities. The interpretation of this result is less clear. Apparently disconnected conditions related to the appearance of a non-Kolmogorovian behavior were proposed by Aerts (1986) and Czachor (1992). We propose here a sufficient condition to be fulfilled by hidden variable theories in order to allow them to be non-Kolmogorovian. We discuss the distinction made by Aerts between hidden state variables and hidden measurement variables, as well as the criterion of Czachor in the light of our new condition, and show how it is possible to conciliate their apparently orthogonal points of view thanks to our new condition. We propose an experimental setup aimed

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[2] For instance, the Gutkoski–Masotto (1974) inequalities express that probabilities which appear in a Bell-like situation admit a Kolmogorovian representation. They are equivalent to Bell's (1965) inequalities. The work of Gutkoski and Masotto was generalized by Accardi and Fedullo (1982) and Pitoski (1989). The Accardi–Fedullo (1982) inequalities express the same condition as those of Gutkoski and Masotto (1974) in terms of conditional probabilities. Pitoski's (1989) inequalities apply to a Bell-like situation with four different directions for the polarizers instead of three, as is the case in the original Bell's (1965) theorem, and they lead to the Clauser–Horne (1974) inequalities.

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at measuring eventual memory effects associated with hypothetical hidden measurement variables.

1. THE ACCARDI–FEDULLO INEQUALITIES

In 1982, Accardi and Fedullo deduced inequalities which express the existence of a Kolmogorovian model for conditional probabilities, and showed that the quantum probabilities do not admit such a model. Let us now present a summary of their work.

1.1. The Kolmogorovian Model for Three Conditional Probabilities

Let us consider three dichotomic experiment A, B, C with the outcomes A_+, A_-, B_+, B_-, C_-, C_+ and the conditional probabilities between them. Usually, they are presented in a matrix of which the element $P(X|Y)$ represents the probability of observing the event X when the event Y is realized with certainty 1. This matrix possesses 36 elements, but $P(X_{+/-}|X_{+/-}) = 1 = 1 - P(X_{-/+}|X_{+/-})$, so that only 24 of them are unknown *a priori*. The conditional probabilities are said to admit a Kolmogorovian model iff:

- There exists a probability space Ω with a measure μ[3] on it.
- To each experiment, we can associate a measurable partition of Ω (for instance, for A, we have A_+, A_-: $A_+ \cap A_- = \emptyset$, $A_+ \cup A_- = \Omega$).
- The conditional probability is given by the Bayes formula: for instance, $P(A_+|B_+) = \mu(A_+ \cap B_+)/\mu(B_+)$.

The possibility of existence of a Kolmogorovian model is the object of a theorem of Accardi and Fedullo (1982):

Theorem 1. If the conditional probability is symmetrical, so to say, if $P(X_+|Y_+) = P(Y_+|X_+)$ and $P(X_-|Y_+) = P(Y_+|X_-)$, it admits a Kolmogorovian model iff the three conditional probabilities p, q, r [respectively $P(A|B)$, $P(B|C)$, $P(C|A)$] fulfill the inequalities

$$|p + q - 1| \leq r \leq 1 - |p - q| \tag{1}$$

1.2. Violation of the Inequalities by Quantum Probabilities

As was noticed by Accardi and Fedullo (1982), the quantum probability related to a Stern–Gerlach spin-1/2 measurement violates the inequalities, as shown by the following example. Let us represent a spin-1/2 state by the

[3] This implies among other results that the measure is always positive, that the measure of Ω is 1, that the measure of the union of two disjoint sets of Ω is the sum of their measures, and that if $A \subset B$ in Ω, then $\mu(A) \leq \mu(B)$.

associated Bloch vector: $(x, y, z) = (\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle)$, where $\langle\sigma_i\rangle$ represents the average value of the i-Pauli matrice associated to a spin measurement with the Stern–Gerlach magnet along of the i direction (i: x, y, z). Pure states are mapped by this transformation onto the unit sphere, so that we can equivalently represent a pure state by a point of the surface of this sphere. Let us make the following choice for A, B, C: let us identify A, B, C with three points belonging to the same great circle on the sphere, A making with B an angle $\pi/3$, C making with A an angle $2\pi/3$, and with B an angle $\pi/3$. The quantum probability of transition from, let us say, A to B is, according to standard quantum computations, equal to $\cos^2(\theta_{AB}/2)$, where θ_{AB} is the angle taken on the sphere between the points A and B. Then, our choice for A, B, and C implies that $p = 3/4$, $q = 3/4$, $r = 1/4$, and the Accardi–Fedullo inequalities, necessary for the existence of a Kolmogorovian model, are violated. For instance, $|p + q - 1| = 1/2 \nless 1/4 = r$, in contradiction with the required inequality.

2. A SUFFICIENT CONDITION FOR THE VIOLATION OF THE INEQUALITIES

Apparently, the inequalities of Accardi and Fedullo show an antinomy between the classical probability and the quantum probability. The fact that probabilities appear in quantum experiments, for instance, in Stern–Gerlach experiments, which do not admit a Kolmogorovian model in the sense of Accardi and Fedullo (take three coplanar directions for the apparatus, the second and the third of them making an angle of $\pi/3$ with the foregoing), could be considered as an experimental proof that the probabilities which we observe in nature are not explainable in terms of simple models. This was sometimes considered as a no-go theorem against the existence of hidden variable models aimed at simulating quantum probabilities. Unfortunately for the partisans of this line of thought, such models exist, and they do not necessarily require the introduction of exotic, non-Kolmogorovian probabilities.[4] We shall now give such a counterexample, which makes it possible to simulate very easily quantum probabilities in the case of a Stern–Gerlach measurement.

2.1. A Simple Model

Let us assume that the spin state in which we prepare the system is represented by a point A on the sphere, and that we measure the spin with the Stern–Gerlach magnet along the direction B. We associate to this situation

[4]This possibility was investigated by Pitoski (1982), Accardi (1984), and Gudder (1984).

a random hidden variable x which is homogeneously spread over the real interval [0, 1], and impose the following rule. During each measurement process, the value of x is fixed and its value determines unambiguously the result of the measurement according to the following requirement: when $x < \cos^2(\theta_{AB}/2)$, where θ_{AB} is the angle taken on the sphere between the points A and B, the result of the measurement is spin up, otherwise it is spin down. When we average these results over a large number of experiments, we recover the standard quantum probability, $\cos^2(\theta_{AB}/2)$ for spin up, $\sin^2(\theta_{AB}/2)$ for spin down.

2.2. A Sufficient Condition for Violating the Inequalities

Note that the probability distribution of the hidden variable (homogeneous distribution over a real interval) is a very "brave" and regular one which fulfills obviously Kolmogorov's definition (a measure on a σ-algebra). There is also no problem with the application of Bayes' law, which we use in a very natural way when we divide the length of the segment $[0, \cos^2(\theta_{AB}/2)[$ by the length of the total segment [0, 1]. What is then the ingredient in our model which allows us to overcome the constraints imposed by the inequalities? According to us, our model allows us to simulate quantum (and thus non-Kolmogorovian) probabilities for the following reason, which we shall from now on call condition B, in order to differentiate from the Aerts (1986) and Czachor (1992) conditions, which we shall call conditions A and C, respectively.

Condition B. A hidden variable model fulfills condition B when the specification of the hidden variable only is not sufficient to predetermine the result of a measurement, unless we also specify the initial state in which the system is prepared.

In the simple model of the previous section, for instance, for the same value of the hidden variable $x = 1/2$, the result of a spin measurement with the Stern–Gerlach magnet along the direction A is spin up (down) according to the fact that we prepare the initial state respectively in B or C. This is sufficient in order to violate the inequalities because this type of hidden variable model is not covered by the assumptions made by Accardi and Fedullo in their definition of a Kolmogorovian model. Effectively, there is only one straightforward way to associate a hidden variable model to the Kolmogorovian model given in the previous section; it is to associate to every subset $X \cap Y \cap Z \ldots$ of Ω (where X, Y, Z belong to $\{A_+, A_-, B_+, B_-, C_-, C_+\}$) a hidden variable which will be realized with probability $\mu(X \cap Y \cap Z \ldots)$, and in which the results X, Y, and Z are predetermined with probability 1. The simple hidden variable model that we presented here cannot be put in this category of models because in our case we must also specify the angle between the conditioned initial state and the final state in order to know the result of the measurement interaction. This is why we can overcome the limitations imposed by the inequalities of Accardi and Fedullo: Accardi and Fedullo implicitly assumed in the hypothesis of their theorem that the result of a measurement is independent of the conditioning, but fixed once for all (the measurement is an observation, and the conditioning does not change the distribution μ in Ω, it just focuses on a subdistribution). Our simple model of Section 2.1 shows thus that it is possible to simulate any kind of probabilities with a sufficiently flexible hidden variable model. As de Broglie said, "no-go theorems just show a lack of imagination."

3. COMPARISON OF OUR SUFFICIENT CONDITION FOR THE VIOLATION OF THE INEQUALITIES WITH AERTS' AND CZACHOR'S CONDITIONS

3.1. The Aerts Condition

Aerts (1986) presented a hidden variable model in which the hidden variable is associated to the measuring apparatus [instead of being associated with the system under measurement itself, as for instance, in Bohm's interpretation (1952)] and proposed this kind of model (which he called "hidden measurement" models, to differentiate them from Bohm-like models, which he called "hidden state" models) as the class of models which can explain the appearance of non-Kolmogorovian probabilities. Before we discuss the relevance of this assertion, let us give concrete examples of a hidden measurement and a hidden state model.

3.1.1. A Hidden Measurement Model: The Elastic Model

In this model (Aerts *et al.*, 1997), the apparatus itself is characterized by a hidden state, and this hidden state undergoes fluctuations which are assumed to be at the origin of quantum stochasticity. The model is defined as follows. The hidden state of the apparatus is represented by an elastic membrane which is placed inside the sphere, along the direction of the Stern–Gerlach magnet (for sure, this is a metaphorical model!). The act of measurement proceeds as follows: first, the particle, represented by the location of its Bloch vector on the sphere, "falls" onto the elastic, orthogonally to it. It is thus located at a distance cos θ from the center of the sphere; afterward the elastic is assumed to break at random somewhere between its two extremities. This has as effect that the particle is projected on the extremity of the elastic membrane, which is, relative to the break, on the same side as the particle. This is assumed to represent the collapse of the particle on this state, and the observation of the corresponding outcome during the measurement. The probability of, let us say, spin up is then given by $(1 + \cos\theta)/2$, which is equal to the quantum probability $\cos^2(\theta/2)$, as it must.

3.1.2. A Hidden State Model: The Bohm Theory

Bohm's (1952) interpretation is, in summary, the following: whenever we can associate to the equation of evolution of a quantum system a conservation equation of the form $\partial_t\rho = div(\mathcal{T})$, where ρ is a positive-definite density of probability and $div(\mathcal{T})$ is the three-dimensional divergence of a current vector, we can interpret this density as a distribution of localized material points moving with the velocity $\mathcal{T}/\rho$. It is thus possible to formulate a hidden variable theory for the system: it would consist of a spatial distribution of material points which initially coincides with the quantum distribution (given by ρ); these points move with a velocity equal to $\mathcal{T}/\rho$. In virtue of the conservation equation, the spatial distribution deduced from this evolution coincides then for all times with the quantum distribution. According to de Broglie, all the measurements being, in last resort, position measurements, this hidden variable theory is, for practical purposes, equivalent with orthodox quantum mechanics.

In the case of a single particle passing through a single Stern–Gerlach apparatus, the situation is particularly simple: when a spin-1/2 particle passes through the magnet, the wave packet associated with the spin up (down) component is deviated upward (downward), and, if the magnetic gradient is strong enough, both components become spatially dissociated. They then form two distinct spots on a screen placed across their trajectories, as observed in the original Stern–Gerlach experiment. The Bohm trajectories associated with this experimental situation, in the case of Gaussian-shaped incoming wave packets, were presented in Dewndney *et al.* (1988). For such packets, the initial state can be put in the form

$$|\Psi(\mathbf{r}, t)\rangle = \psi(\mathbf{r}, t)\cdot(a|+\rangle + b|-\rangle)$$

where the complex numbers a and b are the amplitudes of the up (+) and down (−) spin states. When the incoming wave packet is Gaussian and that the magnet is placed along, say, the Z axis, perpendicular to the incoming velocity v_0 along, say, the axis X, the exact solution of the Pauli–Schrödinger is known (Bohm, 1951). We can then deduce from the equation of conservation (see, for example, Durt 1996a) the following dynamical equations inside the magnet:

$$\frac{dx}{dt} = v_0 + \frac{k^2}{1 + k^2t^2}\,t(x - v_0t)$$

$$\frac{dy}{dt} = \frac{k^2}{1 + k^2t^2}\,ty$$

$$\frac{dz}{dt} = \frac{k^2}{1 + k^2t^2}\,tz$$
$$+ \frac{\left\{-|a|^2 \exp\left(\dfrac{\beta t^2}{1 + k^2t^2}\dfrac{z}{2}\right) + |b|^2 \exp\left(\dfrac{-\beta t^2}{1 + k^2t^2}\dfrac{z}{2}\right)\right\}}{\left\{|a|^2 \exp\left(\dfrac{\beta t^2}{1 + k^2t^2}\dfrac{z}{2}\right) + |b|^2 \exp\left(\dfrac{-\beta t^2}{1 + k^2t^2}\dfrac{z}{2}\right)\right\}}$$
$$\times\left\{\frac{k^2}{1 + k^2t^2}\,\alpha t^3 - 2\alpha t\right\} \tag{2}$$

where k, α, and β are constants, whose values are fixed by the physical magnitudes which characterize the problem (intensity and gradient of the magnetic field, gyromagnetic coupling constant, initial velocity, mass, charge, Gaussian spread in the position of the incoming particle, and so on). These equations can be integrated by computer [a graphical result is given in Dewndney *et al.* (1988)], and the global dynamics can be studied. This work confirms the results that we shall now deduce with the help of the simple following topological argument.

The Bohm trajectories never cross because the Bohmian dynamics is of first order in time. In other words, positions determine velocities, as it is obvious in the previous equations. When the potentials and the initial wave function are separable in Cartesian coordinates, as is the case here, the dynamics factorizes into three dynamics, one for each component. Then, the noncrossing property implies that the trajectory associated with a given initial height z remains "above" all the trajectories associated with initial smaller values of z. If we combine this property with the fact that the Bohmian dynamics preserves the quantum spatial statistics in $|\psi(x, y, z)|^2$, we obtain that the "attractor basin" of the initial positions for which z is larger (smaller) than z_0, where $\int_{z_0}^{+\infty}|\psi(x, y, z)|^2 = \cos^2(\theta/2)$, is the upper (lower) spot,[5] as it must be. Clearly, here the relevant hidden variable is the height of the initial position occupied by the particle inside the wave packet. Obviously, this is a hidden state variable in the sense that it characterizes the hidden state of the quantum system under measurement itself, and not the measuring apparatus.

[5] Exact computations show that between these two extreme behaviors, a fuzzy zone centered around z_0 remains when the screen is placed at finite distance. When initial heights belong to this zone the impact on the screen will be located somewhere between the two spots. When the screen is placed further the distance between the spots increases, while the extent of this fuzzy zone (and thus its probabilistic weight) diminish altogether. This fuzzy zone corresponds to the intersecting tails of the Gaussian packets associated with the up and down components of the outgoing packet.

3.1.3. Comments on Aerts' Hypothesis

These two examples help to understand the distinction between hidden measurement variables (Aerts' case) and hidden state variables (Bohm's case). Note that the initial height plays in Bohm's case a role comparable to the breaking point of the elastic in the elastic model, or to the variable x distributed at random inside the interval $[0, 1]$ in what we called a simple hidden variable model in Section 2. Obviously, by reparametrizing the variable z in Bohm's case, or the breaking point of the elastic in Aerts' case, we arrive at a hidden variable model which is essentially equivalent to this simple hidden variable model, provided we neglect "ideological" criteria such as "position is a preferred variable" in Bohm's case, or "the hidden variable characterizes the apparatus only and not the system under measurement" in Aerts' case.

Note also that in these three models, we must specify the angle θ between the conditioned initial state and the final state in order to know the result of the measurement process. In the elastic model, this is obvious because for the same breaking point the particle can fall "above" or "under" this breaking point, depending on the value of θ. In the Bohmian model, when we rotate the magnet in the, say, YZ plane, perpendicular to the incoming direction chosen here to be the axis X (or, equivalently, when we rotate the Bloch vector of the spin of the incoming particle in the opposite direction), it appears that, for the same initial position, the particle can move up or down, depending on the value of θ. This means that Condition B is satisfied in the three models previously studied. In fact, we checked the litterature about hidden variable models, and Condition B appeared to be fulfilled for all of them, which shows that this condition is really a minimal one.

It is worth recognizing that this condition is, in essence, a nonclassical condition. To show this, let us return to statistical mechanics, which is the classical theory that inspired the promotors of hidden variable theories. In statistical mechanics, the hidden variables are points of the classical phase space (seven real coordinates in the simplest cases: three for the impulsion, three for the position, and one for the time). They are weighted with some initial distribution (for instance, the Boltzmann distribution), and conserve this weight when they evolve in the phase space under the influence of external forces. This type of evolution is, for instance, described by the Liouville equation. What is typical in such approaches is that the knowledge of the initial coordinates and the knowledge of the external forces is sufficient in principle in order to deduce deterministically the subsequent evolution of the "hidden" state, and thus to predict the result of any measurement process undergone by the particle. If Condition B is fulfilled, this is no longer true, because in addition to the knowledge of the hidden state (or hidden variable), we need to specify the quantum state. In Bohm's theory, for instance, the knowledge of the initial hidden state (the position) and of the external forces is not sufficient to predict its subsequent evolution. We must also integrate an extra force, due to the quantum potential, which depends on the quantum state (the whole wave function). Now, the quantum state $\psi(x, y, z)$ contains all the information about the distribution $|\psi(x, y, z)|^2$ of hidden states (the position), so that we could think that the quantum potential expresses the interaction between different particles of same nature, distributed according to $|\psi(x, y, z)|^2$, but the wave function is still valid in a low-intensity regime, for instance, with one particle at a time in the Stern–Gerlach device, so that it is impossible, in the last resort, to find a classical analogy for such a situation.

Now that we have given these concrete examples of a hidden state and a hidden measurement model, we can discuss Aerts' hypothesis (Aerts, 1986) according to which "the nonclassical probability calculus of quantum mechanics can be interpreted as being the result of a lack of knowledge about the measurements," or, in other words, hidden measurement variable models allow us to simulate quantum (non-Kolmogorovian) probabilities. Let us formulate this under a form similar to Condition B:

Condition A. A hidden variable model fulfills Condition A when the hidden variables describe the hidden state of the measuring apparatus only, and not the hidden state of the system under measurement.

The example provided by Bohm's theory, which is a hidden state theory, shows that this condition (of lack of knowledge about the measurement) is certainly not a necessary condition for obtaining non-Kolmogorovian probabilities. Nevertheless, in nearly all interesting cases, hidden measurement models satisfy the sufficient condition for the violation of the inequalities that we gave in Section 2 (Condition B), as we shall show now.

Let us assume that the probability distribution of the outcomes of a given experiment is simulated by a hidden measurement model, and that Condition B is not fulfilled, so that the specification of the hidden variable only is sufficient to predetermine the result of a measurement, disregarding the initial state in which the system is prepared. This implies that the distribution of probability of the different outcomes of the experiment is independent of the initial state in which the system under measurement is prepared. Clearly, this "solipsistic" situation is not an interesting physical situation. Physics aims at finding permanence beyond impermanence, but a situation in which we have no control at all (permanence without impermanence) of the results is rarely interesting. In summary, when a hidden measurement model simulates a probabilistic behavior in which probabilities depend on the initial state of the system under measurement, Condition B is fulfilled. We showed that this condition was sufficient for the violation of the inequalities, so this is also true for Aerts' hypothesis, in nearly all cases.

Note that Condition B is still valid for hidden state models, so it is a more general condition than Condition A. Let us now discuss the sufficient condition formulated by Czachor.

3.2. The Czachor Condition

Czachor (1992) analyzed the charge model given in Aerts (1986), a model essentially equivalent to the elastic model presented here, and arrived at the following conclusion: "As long as the concrete Aerts model is concerned, two assumptions are needed: the conditioning by a polarization and a lack of knowledge about the measurement. It is still unclear for the author of this paper whether the latter condition is sufficient for the non-Kolmogorovity of the description." (Czachor 1992).

We clarified the question contained in the last sentence in the previous subsection. The new element introduced by Czachor is what he elsewhere calls the "polarization effect," or "the conditioning by a change of state," which we express through Condition C:

Condition C. A hidden variable model fulfills Condition C when the measurement process changes the value of the quantum state and/or of the hidden variable during the measurement.

This is a generalization, formulated at the level of hidden variable theories, of the so-called collapse of the wave function. We cannot repeat here all the analysis made by Czachor, but it is clear that, at least implicitly, in Czachor's view, the knowledge of the hidden variable only is not sufficient to predetermine the result of a measurement. In all the hidden variable models of spin measurement studied by him, the knowledge of the angle between the initial Bloch vector of the spin state and the direction of the Stern–Gerlach magnet is necessary in order to predetermine the preferred result. Furthermore, Czachor emphasizes the role played by the measurement process, which in quantum mechanics can be considered as a conditioning, or preparation of the initial state of a subsequent measurement. Condition B is slightly more general than Czachor's, in the sense that we emphasize the role of the initial state, disregarding the technique that we use to prepare this initial state. In summary, the previous discussions show how Condition B provides a bridge between Aerts' and Czachor's approaches: Aerts' condition (hidden measurement) implies ours for nonsolipsistic hidden measurement models, while Czachor's condition implies ours for systems in which the procedure of preparation of the initial state is equivalent to a measurement process, as is the case for quantum systems.

4. HIDDEN STATE VERSUS HIDDEN MEASUREMENT VARIABLES: AN EXPERIMENTAL ANSWER

4.1. The Distinction Between System and Apparatus

The example of a hidden state variable model concretized by Bohm's theory shows that the concept of hidden measurement variables and the concept of conditioning are decoupled, and answers positively to the following question raised by Czachor (1992): "The question whether one is capable of constructing the quantum probability model based on the conditioning by a change of state as the unique non-classical element[6] shall be left open in this paper." We can thus wonder whether the distinction between hidden measurement/hidden state variables is relevant. Is it only a distinction of principle, without any physical relevance? In some sense, this distinction is very close to the ancient distinction between solipsism and realism, which we synthesize as follows. Are the results of our observations, sensations, and so on created by the process of perception, or do they preexist, independently of our observation? Many philosophers have given a very pragmatic answer to the debate, of the kind, "Anyhow, we cannot neglect ourselves and our mind cannot go outside of our consciousness to check that the sources of perceptions are independent from us." This answer is close in some aspects to the Copenhagen view: "Why should we talk about the position of a particle between two successive measurements, because if we do not measure it, we do not know it?" Does our previous discussion about hidden variables help to answer the question? Our sufficient condition emphasized the role played by the initial state and so marks a point in favor of the hidden state approach, but other constraints exist on hidden variable models than Bell-like inequalities (Gutkoski-Masotto, 1974; Accardi and Fedullo, 1982; Clauser and Horne, 1974; Pitoski, 1989). These are impossibility theorems (Bell, 1966; Kochen and Specker, 1967; Belinfante, 1973; Brown, 1992), in which the importance of the role played by the measuring apparatus is emphasized, and made concrete by the new concept of contextuality. We cannot discuss here the role played by contextuality in hidden variables, but undoubtedly it marks a point in favor of the hidden measurement approach. This can also be said about the concept of nonlocality, which we left aside here for reasons of simplicity.[7] Note that, although the role of the initial state may be invoked in order to explain the violation of Accardi's inequalities, it may not be invoked in order to explain the violation of Bell's inequalities because these can be violated by one single well-chosen quantum state.

[6] So to say without introducing necessarily hidden measurement variables.

[7] The interested reader can find in Durt (1997) a discussion of the interrelations which exist between locality and Kolmogorov's axioms, among others, in the framework of Bohm's interpretation.

In conclusion, the new, nonclassical concept of state dependence that we developed here and the new, nonclassical concept of apparatus dependence illustrated by nonlocality and contextuality confirm the pragmatic answer given to the debate solipsism/realism: we cannot neglect, neither the external world nor the internal world, which corresponds here to the system under measurement and to the apparatus. Note that this holistic conclusion is compatible with the Copenhagen as well as Bohmian views. It is just one more confirmation of the good old "way of the middle." Rather surprisingly, this somewhat sterile debate can be expressed in the language of experimenters, provided we make one more assumption about the nature of hidden variables, as we shall show in the next section.

4.2. A Crucial Experiment About Hidden State Models

Over 30 years ago, Papaliolos (1967) realized an experiment aimed at testing the existence of hidden variables described in the Bohm and Bub (1966) model. When this model, which describes the interaction between a quantum system and a measuring device, is applied to the case of a spin-1/2 measurement, it can be shown (Belinfante, 1973) to be essentially[8] equivalent to the simple model presented in Section 2.1. Papaliolos assumed that the Bohm–Bub theory was aimed at describing the passage of a photon through a polarizing device, and made a supplementary assumption: he considered (following Bohm and Bub, 1966) that the hidden variables which hypothetically determine the result of a measurement in the Bohm–Bub theory do not randomize instantaneously, but remain "frozen" during a typical time τ_R. He let low-intensity light pulses prepared in a given polarization state pass through two successive, polarizers that were very close to each other. If the distance between these two polarizers is smaller than the randomization time of the hidden variable times the speed of light, new correlations are predicted by the Bohm–Bub theory (departure from the Malus law), as we show in the appendix. Papaliolos did not see any such effect even for extremely short distances, and considered this negative result as proof of the nonexistence of Bohm–Bub-like variables.[9]

Tutsch (1989) later criticized the relevance of these negative results by noting that, after all, it could be that the hidden variable is not attached to the quantum system only, but that its behavior (and thus its randomization) depends also on the measuring apparatus, in which case the experiment of

[8] If we make abstraction of specific features of the Bohm–Bub model as the description of the collapse process by a nonlinear, continuous-in-time dynamics as the explanation of the randomization of hidden variables in terms of thermal processes, and so on.

[9] Of course, the existence of hidden variables with an extremely short, undetectable memory time is not excluded by experience (Cerofolini, 1982), but the existence of such variables constitutes an ad hoc hypothesis, totally unfalsifiable!

Papaliolos is not conclusive. This is the embryo of the idea of hidden measurement variables, and serves perfectly our present purposes. This intuition of Tutsch is confirmed in the appendix, where we show that, if the hidden states of both polarizers in Papaliolos' experiment are described by independent hidden variables, the statistics of the result is in accordance with the Malus law. The Papaliolos experiment can thus be considered, provided we accept the necessity of the existence of a nonnegligibly small memory time for the hidden variables, not as a crucial experiment which discriminates between hidden variable theories on one hand and the standard interpretation of quantum mechanics on the other, but rather as a crucial experiment which discriminates between hidden state theories on one hand and, on the other hand, the standard interpretation of quantum mechanics and hidden measurement theories. As we shall show now, it is possible to conceive of an experiment which discriminates between hidden measurement theories on one hand and, on the other hand, the standard interpretation of quantum mechanics and hidden state theories.

4.3. A Crucial Experiment About Hidden Measurement Theories

To conceive this experiment, it is sufficient to reconsider Papaliolos (1967) experiment, and to commute in it the role played by the system under measurement (cf. hidden state variables) and the measuring apparatus (cf hidden measurement variables). We must thus consider the probability of, for instance, measuring successively two photons with the same polarization inside the same polarizer. The combined assumption of the existence of hidden measurement variables and of a memory time for them leads to the prediction of results which differ from standard predictions (see Appendix).

5. CONCLUSION

The experiments described in the previous section not only help to bring on the field of experience the old polemics between partisans and detractors of hidden variable theories. Beyond the technicalities inherent to the theoretical considerations developed here, they also make it possible to put directly to nature a very general question, which motivated the title of the present work, and can be formulated as follows: "Do measurable correlation times exist inside the quantum signal?" This kind of question is not so crazy if we think of the mysterious process of building of an interferometric pattern (for instance, in a double-slit experiment). All particles seem to arrive at random on the screen, but, after some time, a structure emerges, the interferometric pattern. It is natural for a physicist who sees order emerging from chaos to try to put into evidence the existence of a "guiding force." Considered so, the existence of memory effects inside the elaboration of the quantum signal is a possible answer to the naive question, "How, by Jove, do the particles know that they must build this pattern?" To be sure, to paraphrase a famous statement of Laplace, we do not need such hypothesis, but the study of correlation times appeared to be fruitful in many scientific research fields (chaotic dynamics, turbulence and so on), and it could be useful in the study of the temporal creation of a quantum interferometric pattern.

Some "sulfurous" theories, as the nonergodic interpretation of Buonomano (1986) and the shape wave theory of Sheldrake (1985), contain hypotheses which go in the same sense; they assume that perhaps the particles are in some way informed of the contribution of the previous particles to the elaboration of the pattern.

Two experiments have attempted to test the possibility of a temporal irreversibility inside the quantum signal: the Papaliolos experiments, aimed at testing the relevance of Bohm and Bub's theory, and Summhammer's experiments [see Buonomano (1989) for analysis and comments], aimed at testing the relevance of Buonomano's interpretation. The experiments of Papaliolos with light pulses excluded the possibility of the hidden state interpretation, and the experiments of Summhammer with neutrons excluded the possibility of the nonergodic interpretation, but other ways stay open, among others the possibility of hidden measurement variables, through experiments similar to the one that we discuss in detail in the second part of the Appendix. An analogue of this experiment is presently in realization at Paris Nord (Laboratoire de Physique des Lasers, J. Robert, *et al.*), in an experimental configuration where a two-level system is concretized by atomic spins. We are presently working on the treatment of numerical data collected during the experiment. In the last resort, as always in science, experiments will decide.

APPENDIX. THE NONSTANDARD PREDICTIONS OF HIDDEN STATE AND HIDDEN MEASUREMENT APPROACHES

A.1. Nonstandard Predictions of Hidden State Approaches

As shown in Belinfante (1973), the Bohm–Bub model in the case of a quantum system described in a two-dimensional Hilbert space is essentially equivalent to the simple model that we described in Section 2.1. We shall use this simple model in order to deduce nonstandard predictions in the case of the Papaliolos experiment. The results appear to differ slightly from the Papaliolos' predictions because the geometry of the distribution of hidden variables in the Bohm–Bub theory is somewhat more complicated than ours. Nevertheless, the concepts involved are the same, so that, for reasons of simplicity, we shall limit ourselves to the simple model. Let us assume that a light pulse is moving along an axis X in a conventional reference frame, and that its state of linear polarization is prepared thanks to a first linear polarizer whose axis of polarization is vertical (parallel to the direction Z of this frame). Then we let the pulse pass through two successive linear polarizers. The axis of the first of them makes an angle θ with the axis Z inside the YZ plane, while the axis of the second is vertical. A filtering in linear polarization is similar to what occurs during a Stern–Gerlach measurement, except that there the probability was $\cos^2(\theta/2)$, while here it is $\cos^2\theta$, in accordance with the classical Malus law, which claims that a fraction $\cos^2\theta$ of the pulses passes through the polarizer. We are here in a low-intensity regime, so that we can consider that the majority of the nonempty pulses contains one photon. Let us assume that our simple model is applicable, that the hidden variable is a property of the pulse itself, and that it remains frozen during the time of flight between the two last polarizers. Then, if the pulse passes through the second polarizer, the hidden variable belongs to the interval $[0, \cos^2\theta[$. But this condition implies that the pulse will pass through the third polarizer, too. So the fraction of pulses which pass through the two last polarizers is $\cos^2\theta$. Standard quantum mechanics predicts that this fraction is equal to $\cos^4\theta$. We also get this result if we assume that our simple model is applicable and that the hidden variable is a property of the polarizers themselves (hidden measurement approach), because then the hidden variables in both polarization processes are no longer the same (even no longer correlated).

A.2. Nonstandard Predictions of Hidden Measurement Approaches

Let us now consider the following experimental situation. Light pulses are moving along the X axis in a conventional reference frame, and we prepare their states of linear polarization using a first linear polarizer whose axis of polarization is vertical (parallel to the Z direction of this frame). Then we let the pulses pass through a linear polarizer whose axis makes an angle θ with the Z axis in the YZ plane. Let us assume that the simple model of Section 2.1 is applicable, that the hidden variable is a property of the polarizer itself, and that it remains frozen during the time which separates two successive pulses. Then, if a first pulse passes through the last polarizer, the hidden variable belongs to the interval $[0, \cos^2\theta[$. But this condition implies that a second pulse will pass through this polarizer, too. So the fraction of pairs of pulses (inside a sufficiently reduced temporal window) which pass through the last polarizer is $\cos^2\theta$. Standard quantum mechanics predicts that this fraction is equal to $\cos^4\theta$. We also get this result if we assume that our simple model is applicable and that the hidden variable is a property of the pulses themselves (hidden state approach), because then the hidden variables in both polarization processes are no longer the same (even no longer correlated).

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